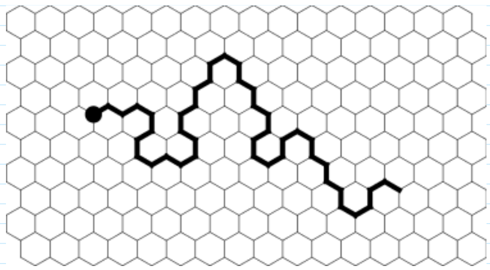
Self Avoiding Random Walk

Paul Flory (1910-1985)

Self-Avoiding Random Walk (SAW) was introduced by Paul Flory (Nobel Laureat, Chemistry, 1974), in 1949. A model for polymers.



Fix a lattice. Our main example: honeycomb lattice in the plane.

Fix a vertex a and consider all non-selfintersecting paths on the lattice starting at a of the fixed length n. Denote the number of them by c_n.

Physics predictions: (for planar lattices!) (Nienhuis, 1982)

1) $c_n \asymp \mu^n n^{11/32}$, up to a constant

μ - lattice dependent - connectivity constant for the lattice.

2) Parametrize the random path as $\omega(nt)$, $0 \le t \le 1$. Then $X(t) = \lim\limits_{n\to\infty} n^{-3/4}\omega(nt)$ exists and has the law of $SLE_{8/3}$.

For comparizone, if $\omega(nt)$ - standard random walk with n step, $B(t) = \lim n^{-1/2}\omega(nt)$ - 2D Brownian motion

3) $X(t)$ is obtained from 2D Brownian Motion by taking outer boundary

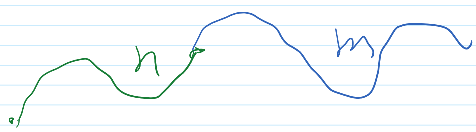
What is the connectivity constant for hexagonal lattice?


Hugo Duminil-Copin

Theorem (Duminil-Copin, Smirnov, 2011)
$\mu = \sqrt{2+\sqrt{2}}$ for the hexagonal lattice.

Remark. Why does μ exist?

Lemma (Standard) If $0 \le a_{m+n} \le a_m + a_n$ then
$\exists \lim_{n\to\infty} \frac{a_n}{n} = \inf \frac{a_n}{n}$.



Observe $c_{n+m} \le c_n c_m$ (can obtain any path of length $n+m$ by first running path of length n and adding a path of length m)

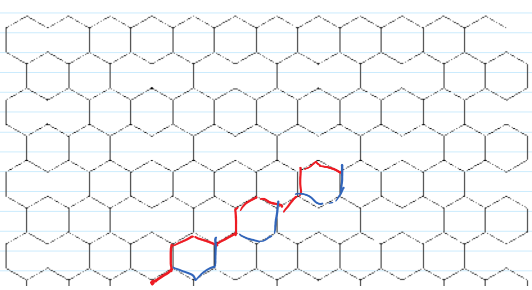
Apply Lemma to $a_n = \log c_n$ to get that

$\log \mu = \lim_{n\to\infty} \frac{1}{n} \log c_n = \inf \frac{1}{n} \log c_n$ exists

$$\mu = \lim_{n\to\infty} \sqrt[n]{c_n}.$$

Observe: $\mu > 0$

Indeed:



$c_{4n} > 2^n$